The Doppler Effect of a Sound Source Moving in a Circle

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When I first heard about an experiment that illustrates the Doppler effect by twirling a buzzer around one's head,[1] I was intrigued and wondered if this experiment would work or not. In fact, it works because you usually do not place both ears exactly in the center of the circle; this would be very difficult indeed. When the source moves in a circle, the distance from the center is always constant, and there is no approaching or receding velocity at all. Holding a microphone in the same hand that holds the string that is attached to the buzzer easily shows this simple fact. No Doppler effect is observed. Although several qualitative demonstrations and experiments of the Doppler effect have been published,[1,2] only a very few quantitative experiments are found in the literature.[3] Having devised a new method to quantify the Doppler effect, described in a previous article,[4] we thought it would be a good physics and geometry exercise to calculate and measure the frequency variations when the microphone is placed on the circular path of the buzzer.

Experimental Setup

In order to do that, the apparatus shown in Fig. 1 was set up. A motor controlled by a variable dc power supply spins a 1.5-m long aluminum bar. On this bar a counterweight and a buzzer connected to a 9-V battery are fixed.

The bar is set to spin at a constant angular speed and the sound is recorded. The sound of the buzzer at rest must also be recorded as a reference. The sound is captured as a .wav file using a microphone connected
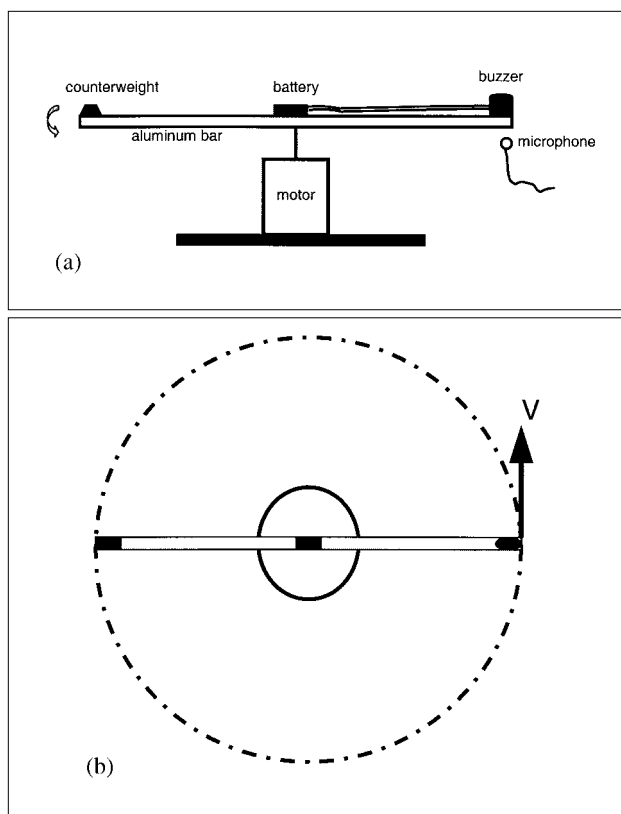

Fig. 1. Schematic of the experimental setup: (a) side view, (b) top view.

to the "line in" computer's soundboard, available in most computers.

The .wav file is then analyzed in a spectrogram software program. The software used in our study is available on the web.[5] It displays the audio signal as a frequency-versus-time plot with signal amplitude at
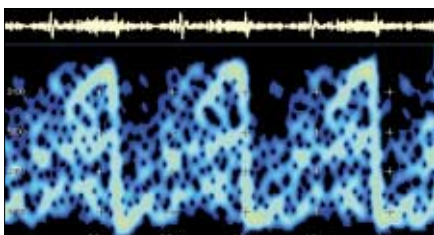
Fig. 2. Spectrogram of the recorded sound of the moving source (below) and signal intensity (above).
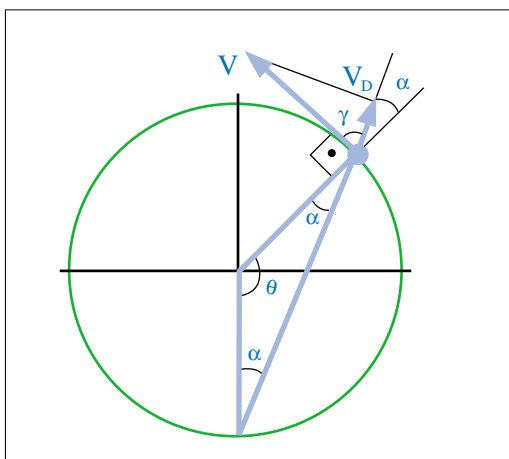

Fig. 3. Schematic geometry to calculate the approaching and receding speeds.

each frequency represented by intensity (or color). Also, a continuous readout of time (ms), frequency (Hz), and signal level (dB) at the position of the mouse pointer (cursor) allows an easy sampling of the frequencies with maximum signal level.

Figure 2 shows a typical display given by the spectrogram software. The recorded frequency of the buzzer increases to its maximum value when it approaches the microphone (maximum approaching speed). And it is abruptly decreased to its minimum just after the buzzer passes over the microphone (maximum receding speed). When the buzzer is located opposite to the microphone, there is no movement towards or away from it. Therefore, the recorded frequency (f) is the same as that produced by the buzzer at rest (f_0).

Theory

Let the receding/approaching velocity be V_D. It will be positive if the buzzer is receding and negative if

it is approaching the microphone. If the observer (microphone) is at rest, the relationship between f and f_0 will be given by the well-known Doppler effect equation:

$$f = f_0 \left(\frac{v_s}{v_s + V_D} \right), \tag{1}$$

where v_s is the sound velocity.

In order to find V_D as a function of time, one needs to make some geometrical considerations based on Fig. 3.

$$V_D = V \cos \gamma,$$

with V the tangential velocity of the buzzer and γ the angle between V and V_D. If T is the period of rotation of the buzzer and R the radius of the circle, then

$$V = \frac{2\pi R}{T},$$

which gives

$$V_D = \frac{2\pi R \cos \gamma}{T}. \tag{2}$$

Observing the angles in Fig. 3 we find that $\theta + 2\alpha = \pi$ and $\alpha + \gamma = \pi/2$. Solving for γ we get $\gamma = \theta/2$. For a constant V, $\theta = 2\pi t/T$, which makes $\gamma = \pi t/T$. Direct substitution into Eq. (2) yields:

$$V_D = \frac{2\pi R}{T} \cos \left(\frac{\pi t}{T} \right). \tag{3}$$

Substituting this expression into Eq. (1) gives the final equation:

$$f = f_0 \left[\frac{v_s}{v_s + \dfrac{2\pi R}{T} \cos \left(\dfrac{\pi t}{T} \right)} \right]. \tag{4}$$

Results

We can determine T dividing the time of 10 revolutions by 10 or looking directly at the spectrogram to find the time between two consecutive peak frequencies in Fig. 2. From the spectrogram of the buzzer at rest, we can find f_0.
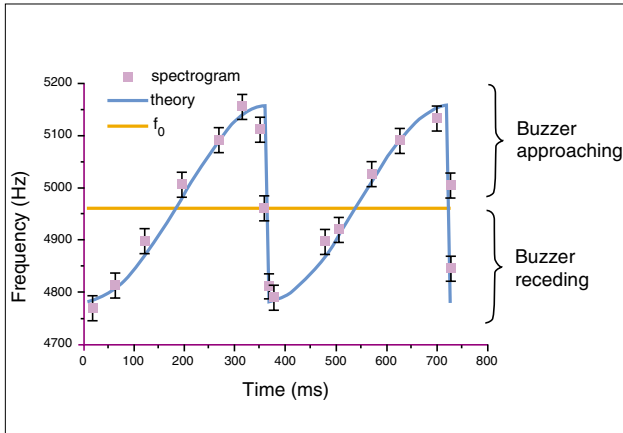
Varying t from 0 to T for two revolutions, we su-

Fig. 4. Calculated and recorded frequency variations.

References

1. P. Doherty and D. Rathjen, *The Exploratorium Science Snackbook* (San Francisco, 1991), p. 41-1.

2. V. Mallete, "Doppler effect using a high frequency buzzer," *Phys. Teach.* **10**, 283 (May 1972).

3. R. Gagne, "Determining the speed of sound using the Doppler effect," *Phys. Teach.* **34**, 126 (Feb. 1996).

4. M.M.F. Saba and R.A.S. Rosa, "A quantitative demonstration of the Doppler effect," *Phys. Teach.* **39**, 431–433 (Oct. 2001).

5. GRAM software, by R. S. Horne, available at: http://www.monumental.com/rshorne/gramdl.html. Find links to other commercial and freeware programs for audio spectrum analysis at http://www.monumental.com/rshorne/links.html.

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perimposed the theoretical Eq. (4) on the experimental data taken from the spectrogram (see Fig. 4). The solid line in the figure is the theoretical curve Eq. (4). The data points are represented by small squares and the error bars are also shown. Experimental results are in good agreement with theoretical predictions.

Comments

The experiment presented here provides nice additions to the standard study of the Doppler effect in a straight-line movement.[4] We found that, apart from having a good chance of applying more geometry and trigonometry to physics, this relatively simple experiment gives a deeper understanding of the Doppler effect. This was very clearly observed when we saw that the students could tell the position of the buzzer in each moment of the frequency-versus-time graph (Fig. 4).

Acknowledgments