Fourier Analysis in Introductory Physics

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In an after-dinner talk at the fall 2005 meeting of the New England chapter of the AAPT, Professor Robert Arns drew an analogy between classical physics and Classic Coke. To generations of physics teachers and textbook writers, classical physics was the real thing. Modern physics, which in introductory textbooks "appears in one or more extra chapters at the end of the book, ... is a *divertimento* that we might get to if time permits." Modern physics is more like vanilla or lime Coke, probably a fad, while "Classic Coke is part of your life; you do not have to think about it twice."

Why have physics teachers focused on classical physics while they are fully aware of the impact of modern physics on society and of the interest students bring to the subject? One of the reasons is the mathematics that is required for the serious introductory course—namely *calculus.*

The problem is that calculus is conceptually inconsistent with quantum mechanics. In the calculus description of motion we have the limiting process where we define instantaneous velocity and acceleration by taking the limit as the time Δt between position measurements goes to zero. Yet from the time-energy form of the uncertainty principle we learn that, in this limit, knowledge of the energy of an object becomes completely uncertain.

If you include Fourier analysis as part of the mathematical foundation of your introductory course, this difficulty vanishes. We will show that by using Fourier analysis, the uncertainty principle, particularly the time-energy form, is a clear and natural consequence of the particle-wave nature of matter when combined with Max Born's probability interpretation of the particle's wave.

The MacScope II Program

In this paper we introduce[1] MacScope II, a software oscilloscope program designed to make it easy to use Fourier analysis in any introductory physics course. We have made MacScope II a shareware program, which you own if you have purchased the $10 *Physics2000* CD. The latest version can be downloaded from the Physics2000.com website.

Special features of the MacScope II program are 1) it works on any computer we have seen that has a USB input (both Mac and Windows). 2) The program is inexpensive to use. If your computer has a sound input, no external hardware is needed. You can turn a USB port into a quality sound input with the $40 iMic[2] USB sound input interface and a $9 Wal-Mart microphone. 3) MacScope II functions as an audio oscilloscope with automatic triggering, manual triggering, and built-in signal averaging. 4) MacScope's Fourier analysis program allows you to reconstruct your experimental curve from selected harmonics. 5) In MacScope II we introduce the *Pulse Fourier Transform*, which makes it easy to study the harmonic structure of a short pulse. The last two features, which we have not seen in other Fourier analysis programs,[3] are what make MacScope II an effective tool for teaching the time-energy form of the uncertainty principle.

Included in the MacScope II instruction manual are two articles at the level of *The Physics Teacher*. One

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is on teaching Fourier analysis, including a noncalculus derivation of the underlying mathematics. The other is on teaching the uncertainty principle. The articles are also available in PDF form at Ref. 4. Here we will summarize the contents of these two articles.

Teaching Fourier Analysis

In the first paper, "Teaching Fourier Analysis," we describe a project in which we analyze the middle C-note on a grand piano. Figure 1 shows the equipment used, a Wal-Mart microphone, the iMic USB interface, and a portable computer. Figure 2 shows the modes of oscillation of a piano wire and the location of the hammer striking the wire. In doing this project, we discovered that for every note on our grand piano, the hammer struck the piano wire 14% from one end, which is at the node of the seventh harmonic.

Figure 3 shows the MacScope II Fourier analysis of the piano note. We selected one cycle of the note and pressed a button labeled Fourier. That one cycle expands to fill the window (the red, more complex curve), and the Fourier analysis window appears below the curve. The harmonics contained in that cycle are represented by vertical bars with the amplitude of the largest harmonic (here the second) scaled to unity. The interesting physics here is that the seventh harmonic is missing, which is what we expect because the hammer struck at a node of that harmonic.

The bars representing the first two harmonics are black because they were selected by clicking on them. In the oscilloscope window you see the smoother, blue curve, which represents the sum of the two selected harmonics. As you select more harmonics, the recon-

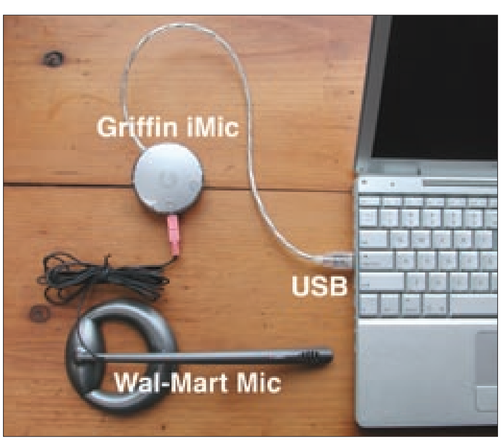

Fig. 1. Apparatus for recording sound waves.

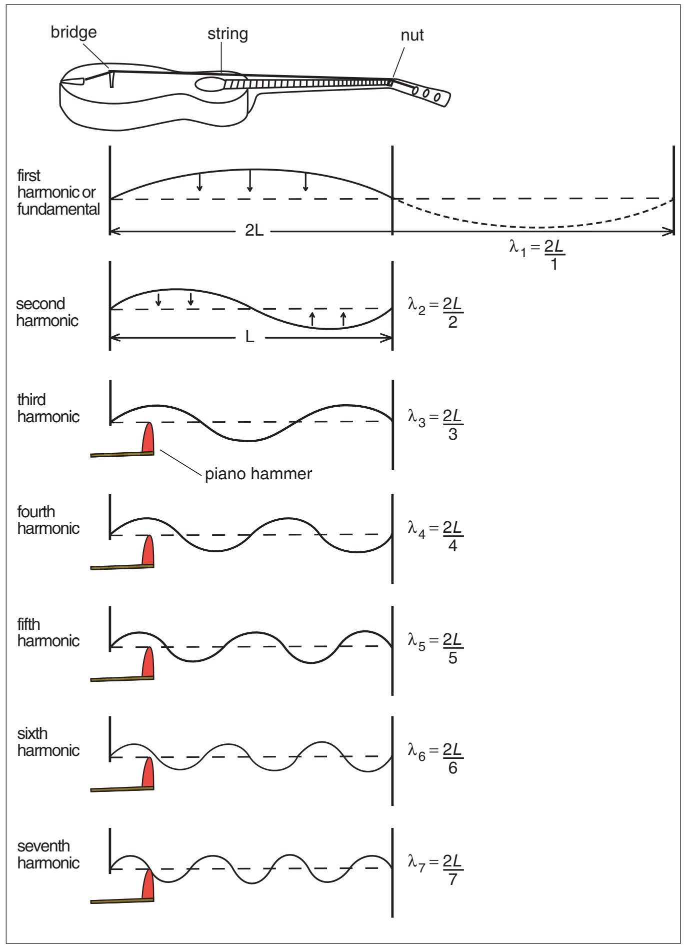

Fig. 2. Modes of oscillation of a piano wire.

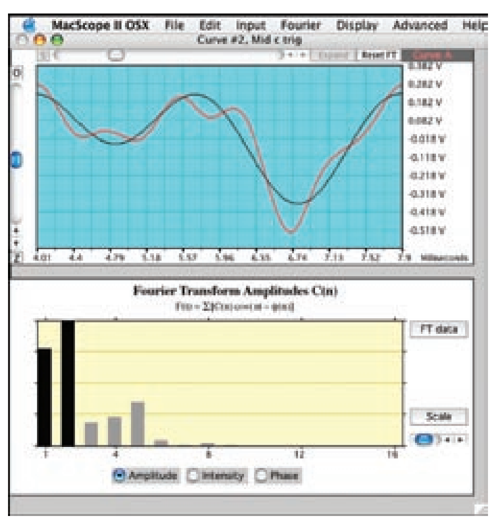

Fig. 3. Fourier analysis window for curve A. When we click on the button labeled Fourier, we get an analysis of the selected section of the curve. Here we see that the seventh harmonic is missing.

structed curve more and more closely approximates the experimental data. By selecting harmonics, you can see precisely how the experimental data is related to its harmonic structure.

Teaching the Uncertainty Principle

In the second paper, "Teaching the Uncertainty Principle," we study the physics of short pulses. Figure 4(a) shows the electric field intensity of a 12-cycle laser pulse.[5] (There are about 24 peaks, because the intensity is the square of the field amplitude.) Figure 4(b) shows the spectrum of wavelengths in the pulse. The spectrum, centered at 800 nm, is in the near infrared. (The visible light spectrum runs from about 400 nm for blue to 600 nm for red.)

What surprises most people is that they think of laser light as being of a pure single frequency. They wonder why this laser pulse has a spread of wavelengths, a spread nearly as wide as the visible spectrum. Does this pulsed laser have a wide spectrum because it is not a very good laser? The answer is NO! A laser pulse as short as the one shown in Fig. 4(a) must have a spectrum as wide as the one shown in Fig. 4(b). We can use MacScope II and Fourier analysis to show you why.

In Fig. 5 we whistled into a microphone to create a sine wave and have selected one cycle of that wave. We then went to the Fourier menu and selected *Pulse Fourier Transform*, a new feature added to MacScope II. What the pulsed Fourier transform does is, instead of expanding the selected section of curve, it zeros the curve outside the selected section to create the pulse seen in Fig. 6. In the Fourier analysis window we get to see the harmonics that are contained in this pulse.

In Fig. 7 we look at the process of reconstructing the pulse from its harmonics. In Fig. 7(a) we selected the center harmonic and see a single sine wave whose wavelength is about the same as our original sine wave but whose amplitude is very small. That one harmonic does not look at all like the pulse.

In Fig. 7(b), we selected five harmonics in the center and see that the harmonics are beginning to add up to create a pulse. They are adding up where the pulse should be and begin to cancel where there is no pulse. In Fig. 7(c) we have selected most of the harmonics and see that the reconstruction of the pulse, as well as the cancellation outside the pulse, is nearly complete.

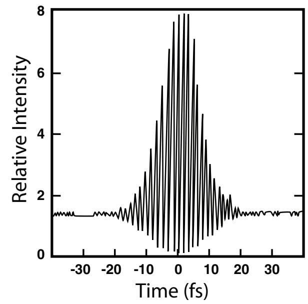

Fig. 4(a). Intensity of the electric field in a 12-cycle laser pulse.

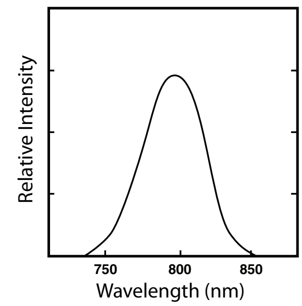

Fig. 4(b). Spectrum of wavelengths in the laser pulse.

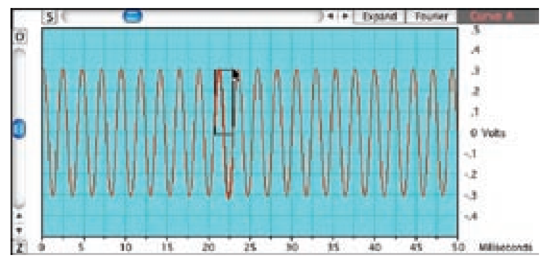

Fig. 5. Selecting one cycle of a sine wave.

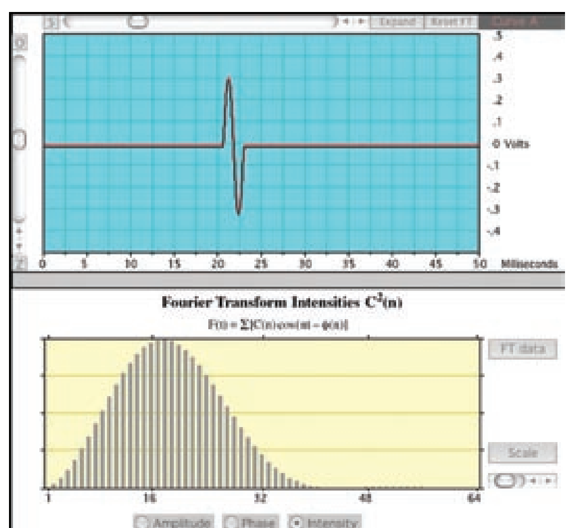

Fig. 6. The Pulse Fourier Transform zeros all but the selected section of the curve, which allows us to study the harmonics contained in a pulse.

What you learn from this is that to construct a pulse from harmonics, from pure sine waves, you need a lot of harmonics to cancel the wave outside the pulse.

In Fig. 8 we went back to our sine wave, selected two cycles, and again chose *Pulse Fourier Transform.* We immediately see that if the pulse is twice as long (two cycles rather than one), the spread in harmonics is half as great (16 harmonics rather than 32). This keeps on going. If you select four cycles, you can reconstruct the pulse with only eight harmonics, etc.

This is a basic property of Fourier analysis. The shorter the pulse, the more harmonics you need to reconstruct it. As a general rule, if you cut the length of the pulse in half, you need twice as many harmonics. As you can verify from MacScope II, this rule depends not on the actual shape of the pulse but mainly on its length.

The Uncertainty Principle

Suppose that our Fig. 7 represented a laser pulse one cycle long. Then what does the Fourier spectrum imply? If there are many photons in the pulse, then the spectrum tells us the distribution of photon frequencies in the pulse. Since the energy of a photon is proportional to its frequency, the spectrum of frequencies is also the spectrum of photon energies. On an appropriate energy scale, the photon energies range from about 1 to 32. (We calculate that energy scale in the article.)

Here is the fun question. Suppose that the pulse contains only one photon. Then how do we interpret the spectrum of frequencies? This is where Max Born's probability interpretation of the particle wave comes in. The intensities of the harmonics are proportional to the probability that the photon has the corresponding frequency or energy. That means that the photon has some probability that its energy is as low as 1 (on our energy scale) or as high as 32. We do not know what the photon's actual energy is, only the probability that it has these different energies. Our knowledge of the energy is uncertain by the width of this spectrum of frequencies or energies.

When we went back and selected two cycles of the sine wave, doubling the length of the pulse, we cut the width of the spectrum in half. Doubling the length of the pulse gives us twice as long to study a photon in the pulse. Cutting the width of the spectrum in half means we have cut in half the uncertainty in our knowledge of the particle's energy. This is the core physics of the time-energy form of the uncertainty principle.

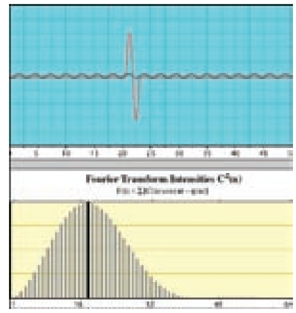

Fig. 7(a). Selecting one harmonic produces a small sine wave, not a pulse.

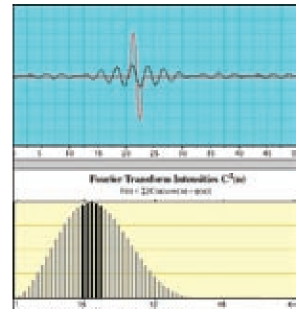

Fig. 7(b). Selecting five harmonics begins to create a pulse.

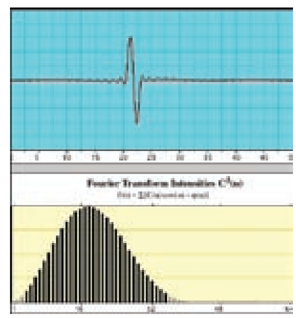

Fig. 7(c). When most of the harmonics are selected, cancellation outside the pulse is nearly complete.

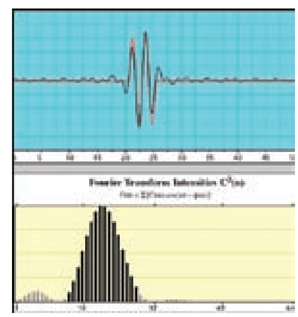

Fig. 8. If we make our pulse twice as long (two cycles), the spread in harmonics is cut in half (from 32 to 16).

References

1. The original MacScope program, which we developed in the 1980s, worked only on the Mac and required external hardware costing nearly $2000. Macscope II is a shareware program available on the *Physics2000* CD and at http://Physics2000.com.
2. iMic is available at http://www.griffintechnology.com.
3. Many data acquisition programs have a Fourier analysis feature, but MacScope's requirement that the user select a repetitive section of a waveform is what allows MacScope to reconstruct that section from selected harmonics.
4. These papers are available at http://ftp.aip.org/cgi-bin/epaps?ID=E-PHTEAH-45-010701. For more information, see the EPAPS homepage, http://www.aip.org/pubservs/epaps.html.
5. F. Hajiesmaeilbaigi and A. Azima, "Ultrashort-pulse generation by self-mode-locked Ti:sapphire lasers without apertures and with low pumping powers," *Can. J. Phys.* **76**, 495–498 (June 1998).

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