Experimenting with Guitar Strings

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What follows is a description of a simple experiment developed in a nonmathematical general education science course on sound and light for fine arts students in which a guitar is used with data collection hardware and software to verify the properties of standing waves on a string.

The resonant frequencies f_n of standing waves on a string obey the expression $f_n = \frac{nv}{2L}$, where n is the mode of vibration or harmonic number, v is the velocity of the waves on the string, and L is the length of string allowed to vibrate. The musical pitch heard when a guitar string is plucked comes from the fundamental $n = 1$ frequency. The higher harmonics, $n > 1$, or overtones, add form to the sound wave heard, which gives the sound of the guitar its unique quality or, as musicians call it, timbre.[1] See Fig. 1 for the waveform produced by a guitar string.

The waveform in Fig. 1 was captured with a microphone attached to a LabPro interface and displayed with Logger Pro.[2] The best results occurred when the microphone was actually placed beneath the strings in the hole in the guitar's body. The time-axis of the display can be used to calculate the period of the waveform that can then be reciprocated to give the frequency.

The frequency and thus musical pitch sounded by a vibrating guitar string is changed when the portion of the string allowed to vibrate is varied. The player holds down the string at different locations along the neck of the instrument, usually with the left hand, and plucks or strums the string with the right hand. Holding down the string on each successive fret on the guitar's neck shortens the string by about 6%, therefore raising the musical pitch by about that much. This is known as a musical half-step, or semitone, the smallest interval used in western music. Note that in the above expression the length of the vibrating string L is in the denominator and will therefore increase the frequency as it is decreased.

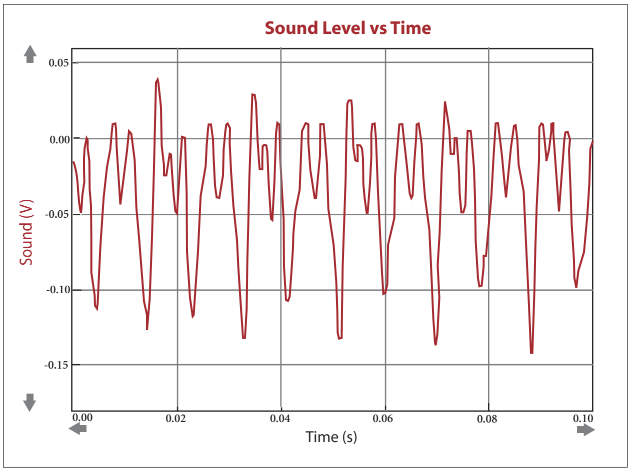

Fig. 1. Waveform created by a vibrating guitar string.

Table I shows the measured lengths of vibrating string and the sound-wave frequencies calculated from the Logger Pro[2] display for each note in an entire musical scale. The E string or "top" string from the player's perspective, which is also the thickest string, was used. The next column shows the frequency produced by each length of string divided by the frequency from the previous length, in most cases increasing by about the expected 6% (a factor of 1.06). The last two col-

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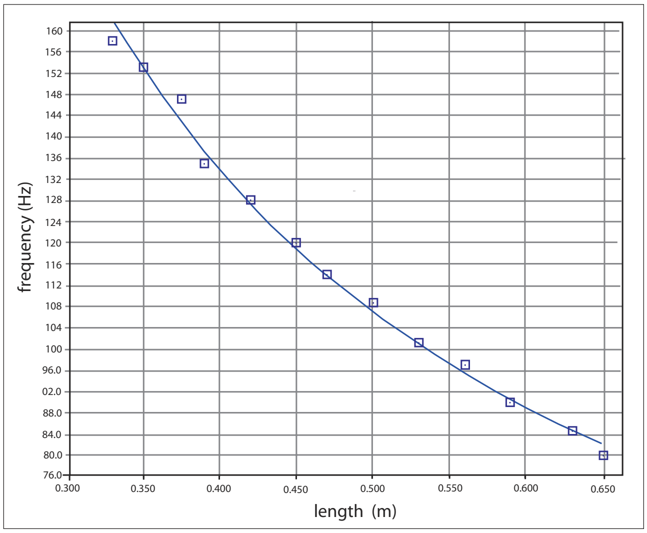

Fig. 2. Plot of frequency vs length of a vibrating guitar string.

Table I. Frequencies of decreasing lengths of a vibrating guitar string compared to frequency of previous length and the entire length.

Length (m)	Frequency (Hz)	*f*-ratio with previous pitch	*f*-ratio with first pitch	expected *f*-ratio with first pitch
0.65	80	1	1	1
0.63	84.7	1.06	1.06	1.06
0.59	90	1.06	1.125	1.122
0.56	97	1.08	1.21	1.2
0.53	101	1.04	1.26	1.26
0.5	108.7	1.08	1.36	1.33
0.47	114	1.05	1.43	1.41
0.45	120	1.05	1.5	1.5
0.42	128	1.07	1.6	1.59
0.39	135	1.05	1.69	1.68
0.375	147	1.09	1.84	1.78
0.35	153	1.04	1.91	1.89
0.33	158	1.03	1.98	2

umns show the ratio that the frequency of each successive pitch makes, with the frequency of the original pitch compared to the frequency ratio of the expected musical interval.

The interval between two musical pitches is the ratio of their frequencies. The expected ratios in Table I come from the pitches of the *equal-tempered* scale currently used in western music and are different than what is known as *just intonation,* on which the frequencies of a standard set of laboratory tuning forks are based. The equal-tempered scale is based on having an *equal* interval between each of the 12 half steps in the musical octave. This is not the case in the just intonation scale. Since an octave has a frequency ratio of 2:1, each half step in the equal-tempered scale has a frequency ratio of $\sqrt[12]{2} = 1.059,$ about 1.06, the 6% increase in frequency from the previous pitch mentioned above.

This became necessary when musical compositions began requiring instruments to change key or to play notes within musical scales beginning on different pitches in the same piece of music. The equal-tempered scale is the only way to ensure that the frequency ratios of the same musical intervals in different keys will be the same.[3]

Figure 2 is a plot[4] of the sound-wave frequencies as a function of vibrating string length from Table I, fit to an equation of the form $f(x)=Ax^B$. The value of B = -1.01 from the curve-fit is very close to the -1 expected from the above expression for standing waves on a string.

The value of A from the fit can be used to calculate the amount of tension on a guitar string. Comparing the fits to the equation, $A = v/2$. The velocity of a wave on a string is given by the expression

$$v = \sqrt{\frac{T}{\mu}},$$

where T is the tension in the string and μ is the mass per unit length in the string. The value of μ can be determined by detaching the string and measuring its mass and complete length. This can also be done with another sample of the same string. The E-string used had a calculated mass/length of μ = 0.0074 kg/m. Using this value of μ and a speed of v = 106.4 m/s calculated from the value of A = 53.2 from the curve-fit gave a tension of T = 83.8 N, which is the equivalent of that much weight or about 8.5 kg ($W = mg$, g = 9.8 m/s^2) of mass being hung from the string.[5] This result is often considered surprisingly large by students, but for comparison, the tensions on piano strings are equivalent to masses of 70–100 kg being hung from them.[6]

Acknowledgments

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References

1. D. Hall, *Musical Acoustics,* 3rd ed. (Brooks/Cole, USA, 2002), p. 181.
2. Logger Pro and the LabPro interface are both by Vernier Software; http://www.vernier.com. Similar hardware and software such as PASCO DataStudio should work as well.
3. For more on this, see Ref. 1, pp. 407–434 or any standard text on musical acoustics.
4. Graphs plotted with Graphical Analysis by Vernier Software, see Ref. 2.
5. D. Hall, "Sacrificing a cheap guitar in the name of science,“ *Phys. Teach*. **27**, 673–677 (Dec. 1989) suggests doing this and notes, as the article's title indicates, that it is potentially destructive to the guitar.
6. I. Johnston, *Measured Tones: The Interplay Between Physics and Music* (IOP Publishing LTD, 2002), p. 75.

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